HEAT TRANSFER FROM A ROTATING CYLINDER UNDER FORCED CONVECTION

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An analytic solution is obtained for the heat-transfer coefficient from a rotating cylinder with inflow of coolant for various Reynolds numbers.

Heat transfer can be intensified by various methods. Heat transfer between rotating coaxial cylinders in the absence of inflow was treated in [1]. In [2] the experimental data were correlated by a buffer model. Recommendations on heat transfer were proposed for a rotating cylinder in a stationary fluid; i.e., in [2] a solution was given as a function only of the Reynolds number for rotation Re_{rot}. The effect of the Reynolds number for inflow Re_{in} was not taken into account, and this led to certain differences from the experimental data. A number of methods for the intensification of heat transfer were correlated in [3], where, because of certain difficulties, the experimental data presented were not correlated with the computational models.

We describe a method for obtaining the change of heat-transfer conditions resulting from the rotation of the heat-transfer surface and inflow of coolant. In the method proposed for calculating the heat-transfer coefficient both inflow and the rotation of the cylinder are taken into account.

We consider a rotating cylindrical heat-transfer surface (Fig. 1) past which a stream of coolant is flowing with a velocity $\mathbf{u}_\infty$. It was shown in [3] that flow past a rotating cylinder can be expressed in terms of the velocity potential

$$\varphi = u_\infty \cos\beta \left(r + \frac{r_0^2}{r}\right) + \frac{\Gamma}{2\pi}\ \beta. \quad (1)$$

To determine the heat transfer from a rotating cylinder we assume, as in [2], that heat transfer occurs completely through the rotating boundary layer, and that the laminar sublayer and the turbulent flow are separated by a buffer layer. We assume, as in [2], that: 1) the buffer layer is located between the relative distances $y^* = 5$ and $y^* = 20$ from the surface of the cylinder; 2) the tangential stress and heat flux in the buffer layer are constant.

The relations for the heat fluxes and tangential stresses are written in the form [3]

$$\tau = (\nu_\mathrm{T} + \nu)\ \frac{dW^*}{dy^*}, \quad (2)$$

$$q = \left(a_\mathrm{T} + \frac{\nu}{P_r}\right)\ \frac{W_k}{\nu}\ \frac{dT}{dy^*}. \quad (3)$$

It is assumed that $a_\mathrm{T} = \nu_\mathrm{T} = 0$ for the laminar sublayer. Then we find from Eqs. (2) and (3) that the temperature drop in the laminar sublayer is determined by integrating Eq. (3) with respect to y^* from 0 to 5, i.e.,

$$\Delta T_1 = 5\ \frac{q}{W_h}\ P_r. \quad (4)$$

To determine heat transfer in the buffer layer $5 \leqslant y^* \leqslant 20$ we find the velocity profile of the boundary layer from (1). To do this we substitute $r = r_0 + y$ in (1). By differentiating the expression obtained with respect to β we find the rate of rotation of the boundary layer

$$W_0 = u_\infty \sin\beta \left(1 + \frac{r_0^2}{(r_0 + y)^2}\right) + \frac{\Gamma}{2\pi(r_0 + y)}. \quad (5)$$

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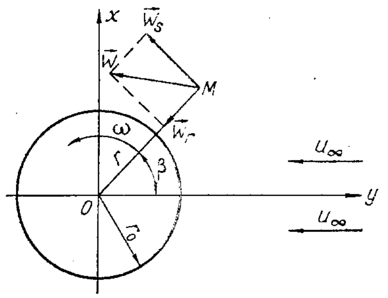

Fig. 1. Flow of coolant past a cylinder.

We calculate the circulation of the velocity Γ in (5) from a plane vortex by the formula [4]

$$\Gamma=\int_L v_x dx+v_y dy=2\,\pi\omega r_0^2 . \tag{6}$$

By substituting (6) into (5) and dividing by W_k we obtain the dependence of W* on the relative displacement y*

$$W^*=\frac{u_\infty \sin\beta}{W_k}\left(1+\frac{r_0^2}{(r_0+y)^2}\right)+\frac{2\,\omega r_0^2}{W_k\,(r_0+y)} . \tag{7}$$

Using the notation

$$\mathrm{Re}_{\mathrm{in}}=\frac{u_\infty r_0}{\nu},\quad \mathrm{Re}_{\mathrm{rot}}=\frac{\omega r_0^2}{\nu},$$

$$W_k^2=c_f\left(\frac{W\mathrm{p}}{2}\right).$$

Equation (7) takes the form

$$W^*=\frac{\mathrm{Re}_{\mathrm{in}}\sin\beta}{\sqrt{\frac{c_f}{2}}\;\mathrm{Re}_{\mathrm{rot}}}\left(1+\frac{\mathrm{Re}_{\mathrm{rot}}^2\,\frac{c_f}{2}}{\left(y^*+\mathrm{Re}\sqrt{\frac{c_f}{2}}\right)^2}\right)+\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*} . \tag{8}$$

Differentiating (8) with respect to y*, we obtain

$$\frac{dW^*}{dy^*}=-\frac{\mathrm{Re}_{\mathrm{in}}\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}\,\sin\beta}{2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*\right)^3}-\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\left(\sqrt{\frac{c_f}{2}}\;\mathrm{Re}_{\mathrm{rot}}+y^*\right)^2} . \tag{9}$$

By substituting (9) into (2) we find the expression for the kinematic coefficient of eddy exchange of momentum

$$\nu_{\mathrm{T}}=-\nu\left(\frac{\mathrm{Re}_{\mathrm{in}}\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}\,\sin\beta}{2(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*)^3}+\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*\right)^2}\right)^{-1}-\nu. \tag{10}$$

In analogy with [2], we write $\nu_T = a_T$ for the buffer layer. Then from Eq. (3) we obtain the following expression for the temperature drop in the buffer layer:

$$dT=\frac{q}{W_k}\;\frac{\left(\mathrm{Re}_{\mathrm{in}}\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}\,\sin\beta+4\,\mathrm{Re}_{\mathrm{rot}}\times\right.}{\left(\frac{1}{\mathrm{Pr}}-1\right)\left(\mathrm{Re}_{\mathrm{in}}\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}\,\sin\beta+4\,\mathrm{Re}_{\mathrm{rot}}\times\right.}\cdots\rightarrow$$

$$\rightarrow\cdots\frac{\left.\times\left(\mathrm{Re}_{\mathrm{rot}}\left(\sqrt{\frac{c_f}{2}}+y^*\right)\right)\right)dy^*}{\left.\times\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*\right)\right)-2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+y^*\right)^3} . \tag{11}$$

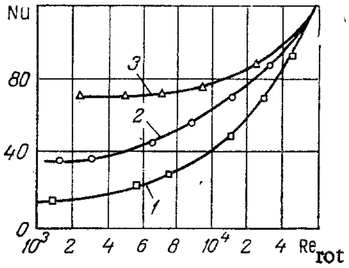

Fig. 2. Heat transfer from a rotating cylinder: 1) calculated according to [2]; 2) calculated from Eq. (19) with $Re_{in} = 1.56 \cdot 10^4$; 3) experimental curve for $Re_{in} = 1.56 \cdot 10^4$ [3].

Integrating Eq. (11) with respect to y* from 5 to 20, and assuming sin β is constant and equal to its average value 0.707, we find the temperature drop in the buffer layer:

$$\Delta T_2 = -\frac{q}{2W_k}\left(A \ln \frac{Re_{rot}\sqrt{\frac{c_f}{2}} + 20 - K}{Re_{rot}\sqrt{\frac{c_f}{2}} + 5 - K} + \frac{M}{2} \ln \frac{\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 20\right)^2 + K\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 20\right) + K^2}{\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 5\right)^2 + K\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 5\right) + K^2} + \frac{2N - MK}{K\sqrt{3}}\left(\operatorname{arctg} \frac{2\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 20\right) + K}{K\sqrt{3}} - \operatorname{arctg} \frac{2\left(Re_{rot}\sqrt{\frac{c_f}{2}} + 5\right) + K}{K\sqrt{3}}\right)\right), \tag{12}$$

where

$$B = \sqrt[3]{\frac{\left(\frac{1}{Pr} - 1\right)}{4} Re_{in} Re_{rot} \sqrt{\frac{c_f}{2}}\, 0{,}707 + \left(1 + \frac{1}{Pr}\right) \times Re_{rot} \sqrt{0{,}45 \frac{Re_{in}^2 c_f}{32} + \frac{8\left(1 - \frac{1}{Pr}\right)}{27} Re_{rot}}};$$

$$C = \sqrt[3]{\frac{\left(\frac{1}{Pr} - 1\right)}{4} Re_{in} Re_{rot} \sqrt{\frac{c_f}{2}}\, 0{,}707 + \left(\frac{1}{Pr} - 1\right) \times Re_{rot} \sqrt{0{,}45 \frac{Re_{in}^2 c_f}{32} + \frac{8\left(1 - \frac{1}{Pr}\right)}{27} Re_{rot}}}; \tag{13}$$

$$K = B + C;\quad A = \frac{4\,Re_{rot} + \frac{Re_{rot} Re_{in} \sqrt{\frac{c_f}{2}}\, 0.707}{K}}{3K};$$

$$M = -A;\quad N = \frac{-Re_{in} Re_{rot} 0{,}707 \sqrt{\frac{c_f}{2}} + AK^2}{K}.$$

Integrating the differential equation (11) from y* = 5 to 20 for Pr = 1, we obtain the following expression for the temperature drop in the buffer layer:

$$\Delta T_2' = \frac{-q}{W_h}\left(\mathrm{Re}_{\mathrm{in}}\mathrm{Re}_{\mathrm{rot}} 0{,}707\sqrt{\frac{c_f}{2}}\,\frac{\left(30\,\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+375\right)}{4\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)^2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5\right)^2}+\right.$$

$$\left.+\,2\,\mathrm{Re}_{\mathrm{rot}}\left(\frac{15}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5\right)}\right)\right). \tag{14}$$

In the layer of secondary flow when y* > 20, the equation for the temperature drop has the form

$$dT = -\frac{q}{W_h}\,dW^*. \tag{15}$$

Integrating the differential equation (15) from $W^*|_{y^*=20}$ to W_p, where W_p is the peripheral velocity of the cylinder, we determine the temperature drop in the layer of secondary flow. Substituting W_0^* from (8) into the expression for W*, y* = 20, gives

$$W_0^* = \frac{0.707\,\mathrm{Re}_{\mathrm{in}}}{\sqrt{\frac{c_f}{2}}\,\mathrm{Re}_{\mathrm{rot}}}\left(1+\frac{\mathrm{Re}_{\mathrm{rot}}\frac{c_f}{2}}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)^2}\right)+\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20}. \tag{16}$$

Thus, we obtain from (15) the following expression for the temperature drop in the layer of secondary flow:

$$\Delta T_3 = \frac{q}{W_h}\left(\frac{W_p}{W_h}-\frac{0{,}707\,\mathrm{Re}_{\mathrm{in}}}{\sqrt{\frac{c_f}{2}}\,\mathrm{Re}_{\mathrm{rot}}}\left(1+\frac{\mathrm{Re}^2_{\mathrm{rot}}\frac{c_f}{2}}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)^2}\right)-\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20}\right). \tag{17}$$

The total temperature drop in the boundary layer is equal to the sum of Eqs. (4), (12), and (17), i.e.,

$$\Delta T = \frac{q}{W_h}\left(5\,\mathrm{Pr}-\frac{1}{2}A\ln\frac{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20-K}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5-K}-\right.$$

$$-\frac{M}{4}\ln\frac{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)^2+K\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)+K^2}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5\right)^2+K\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5\right)+K^2}-$$

$$-\frac{2N-MK}{K\sqrt{3}}\left(\mathrm{arctg}\,\frac{2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)+K}{K\sqrt{3}}-\right. \tag{18}$$

$$\left.-\,\mathrm{arctg}\,\frac{2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+5\right)+K}{K\sqrt{3}}\right)+\frac{W_p}{W_h}-\frac{0{,}707\,\mathrm{Re}_{\mathrm{in}}}{\sqrt{\frac{c_f}{2}}\,\mathrm{Re}_{\mathrm{rot}}}\times$$

$$\left.\times\left(1+\frac{\mathrm{Re}^2_{\mathrm{rot}}\frac{c_f}{2}}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20\right)^2}-\frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}}+20}\right)\right).$$

Taking account of Eq. (18), we write the final equation for the heat-transfer coefficient:

$$
\begin{aligned}
\mathrm{Nu} = \mathrm{Re}_{\mathrm{rot}} \mathrm{Pr} \sqrt{\frac{c_f}{2}} \Bigg/ \Bigg(& 5\,\mathrm{Pr} + \frac{1}{\sqrt{\frac{c_f}{2}}} - \frac{1}{2} A \ln \frac{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20 - K}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 5 - K} - \\
& - \frac{M}{4} \ln \frac{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20\right)^2 + K\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20\right) + K^2}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 5\right)^2 + K\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 5\right) + K^2} - \\
& - \frac{2N - MK}{K\sqrt{3}} \left(\mathrm{arctg} \frac{2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20\right) + K}{K\sqrt{3}} - \right. \\
& \left. - \mathrm{arctg} \frac{2\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 5\right) + K}{K\sqrt{3}} \right) - \frac{0.707\,\mathrm{Re}_{\mathrm{in}}}{\sqrt{\frac{c_f}{2}}\,\mathrm{Re}_{\mathrm{rot}}} \left(1 + \frac{\mathrm{Re}_{\mathrm{rot}}^2 \frac{c_f}{2}}{\left(\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20\right)^2} \right) - \frac{2\,\mathrm{Re}_{\mathrm{rot}}}{\mathrm{Re}_{\mathrm{rot}}\sqrt{\frac{c_f}{2}} + 20} \Bigg),
\end{aligned}
\tag{19}
$$

where K, A, N, and M are calculated according to (13), and for the coefficient of friction it is necessary to use the equations [2]:

for $\mathrm{Re}\sqrt{c_f} > 950$

$$\frac{1}{\sqrt{\frac{c_f}{2}}} = -1.828 + 1{,}77 \ln \mathrm{Re}\sqrt{c_f};$$

for $\mathrm{Re}\sqrt{c_f} < 950$

$$\frac{1}{\sqrt{\frac{c_f}{2}}} = -3{,}68 + 2{,}04 \ln \mathrm{Re}\sqrt{c_f},$$

which were confirmed experimentally in [2]. If Pr = 1, in (19) it is necessary to take $\Delta T_2'$ from (14) instead of ΔT_2 from (12).

In Fig. 2 the solution given by Eq. (19) and that found in [2] are compared with experimental data on heat transfer from a rotating cylinder presented in [3]. The difference between curves 1 and 3 is a result of not taking account of inflow for $\mathrm{Re}_{\mathrm{in}} < 24 \cdot 10^5$. Under actual conditions heat transfer in this range of numbers is very important. The proposed theoretical solution (curve 2) is not essentially different from the experimental data (curve 3). The difference between them results from not taking account of the convective component in the model proposed.

LITERATURE CITED

1. V. K. Shchukin, Heat Transfer and Hydrodynamics of Internal Flows in Fields of Body Forces [in Russian], Mashinostroenie, Moscow (1980).
2. W. Kays, "Heat transfer from a rotating cylinder with and without crossform," Trans. ASME, No. 1, 35 (1958).
3. V. M. Buznik, Intensification of Heat Transfer in Marine Installations [in Russian], Sudostroenie, Leningrad (1969).
4. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1978).